MA2071

Practice midterm

1) (i) State (but do NOT derive) the equation of a plane P through the point (x_0, y_0, z_0) with normal vector $\mathbf{n} = \langle a, b, c\rangle$.

(ii) Given a plane P with equation $ax + bx + cz + d = 0$ and a point $P_1 = (x_1, y_1, z_1)$ *not* on the plane, prove that the distance D from P to P_1 is given by

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

2) (i) Either compute the following limit or prove that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}.$$

(ii) Given $xyz = \cos(x + y + z)$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

3) (i) State Clairaut's Theorem.

(ii) Let $f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Does Clairaut's Theorem apply here? Why or why not?

(iii) Show that any function of the form $z = f(x + at) + g(x - at)$ is a solution of the equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

4) (i) Find all points at which the direction of fastest change of $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

(ii) If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?