

**Name:**

**Student Number:**

**MA2071: Multivariable Calculus**

**Midterm Exam - 100 marks - Show all work**

- 1) (i) State (but do NOT derive) the equation of a plane  $P$  through the point  $(x_0, y_0, z_0)$  with normal vector  $\mathbf{n} = \langle a, b, c \rangle$ .
- (ii) True or False: two planes are parallel if their normal vectors are parallel.
- (iii) Find the distance between the planes  $10x + 2y - 2z = 5$  and  $5x + y - z = 1$  (Hint: are these planes parallel?).

- 2) (i) Either compute the following limit or prove that it does not exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}.$$

- (ii) If  $z = f(x - y)$ , then prove that  $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ .

- 3) (i) Find the angles  $\theta$  in which the directional derivative of  $f(x, y) = x^2 + \sin xy$  at the point  $(1, 0)$  has the value 1.
- (ii) Find the equation of the tangent plane to the surface  $yz = \ln(x + z)$  at the point  $(0, 0, 1)$ .

- 4) Prove that the rectangle with maximum area that has a given perimeter  $p$  is a square.