1) $F(x,y) = x^3 y^4 \underline{i} + x^4 y^3 \underline{j}$

$$f_x = x^3 y^4 \implies f = \frac{1}{4} x^4 y^4 + g_1(y)$$
$$f_y = x^4 y^3 \implies f = \frac{1}{4} x^4 y^4 + g_2(x)$$

$\implies$ Take $f = \frac{1}{4} x^4 y^4$

$\int_C F \, ds$ | $C(t) = (\sqrt{t}, 1 + t^3)$, $t \in [0,1]$

$= f(1,2) - f(0,1)$ | endpoints: $(0,1)$, $(1,2)$

$= 4$

2) $F(x,y,z) = (yz, \; xz, \; xy + 2z)$

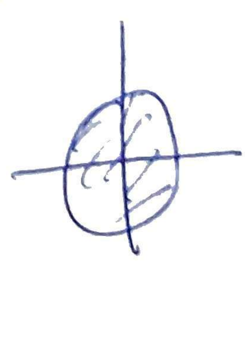
$$f_x = yz \implies f = xyz + g_1(y,z)$$
$$f_y = xz \implies f = xyz + g_2(x,z)$$
$$f_z = xy + 2z \implies f = xyz + z^2 + g_3(x,y)$$

$\implies$ Take $f = xyz + z^2$

$\int_C F \, ds$ C: line segment from $(1,0,-2)$ to $(4,6,3)$

$= f(4,6,3) - f(1,0,-2) = 77$

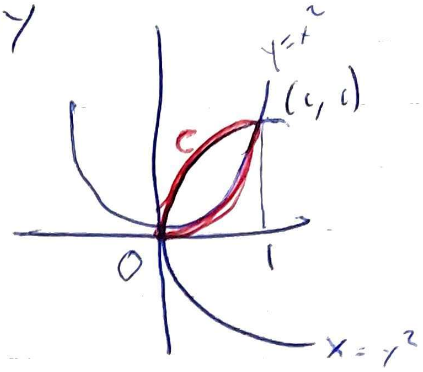
3) $\int_C (3y - e^{\sin x}) \, dx + (7x + \sqrt{y^4 + 1}) \, dy$



$$\int_C P \, dx + Q \, dy = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

$$\hookrightarrow = \iint_D 7 - 3 \, dA = \iint_D 4 \, dA = 4\pi$$

4 $\int_C (y+e^{\sqrt{x}})\,dx + (2x+\cos y^2)\,dy$



$= \iint_D 2-1\,dA$

$= \int_0^1 \int_{x^2}^{\sqrt{x}} 1\,dy\,dx$

$= \int_0^1 \sqrt{x} - x^2\,dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$

5 $\operatorname{curl}(F) = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\underline{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\underline{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\underline{k}$

$\operatorname{div}(F) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

i) $F = xyz\,\underline{i} - x^2y\,\underline{k}$

$\operatorname{curl} = -x^2\,\underline{i} + (xy+2xy)\,\underline{j} + -xz\,\underline{k}$

$\operatorname{div} = yz$

ii) $F = e^x \sin y\,\underline{i} + e^x \cos y\,\underline{j} + z\,\underline{k}$

$\operatorname{curl} F = 0 + 0 + (e^x \cos y - e^x \cos y)\,\underline{k} = 0$

$\operatorname{div} F = e^x \sin y - e^x \sin y + 1 = 1$

6) $\operatorname{curl}(\nabla f) = (f_{zy} - f_{yz})\,\underline{i} + (f_{xz} - f_{zx})\,\underline{j} + (f_{yx} - f_{xy})\,\underline{k}$

$= 0$

7) $\operatorname{div}(\operatorname{curl}(F)) = \cancel{\frac{\partial^2 F_3}{\partial y\,\partial x}} - \cancel{\frac{\partial^2 F_2}{\partial z\,\partial x}} + \cancel{\frac{\partial^2 F_1}{\partial z\,\partial y}} - \cancel{\frac{\partial^2 F_3}{\partial x\,\partial y}} + \cancel{\frac{\partial^2 F_2}{\partial x\,\partial z}} - \cancel{\frac{\partial^2 F_1}{\partial y\,\partial z}}$

$= 0$

8) $F(x, y, z) = 2xy\,\underline{i} + (x^2 + 2yz)\,\underline{j} + y^2\,\underline{k}$

$$f_x = 2xy \implies f = x^2y + g_1(y, z)$$
$$f_y = x^2 + 2yz \implies f = x^2y + y^2z + g_2(x, z)$$
$$f_z = y^2 \implies f = y^2z + g_3(x, y)$$

$\implies f = x^2y + y^2z + C$ is a potential

$\implies F$ is conservative.