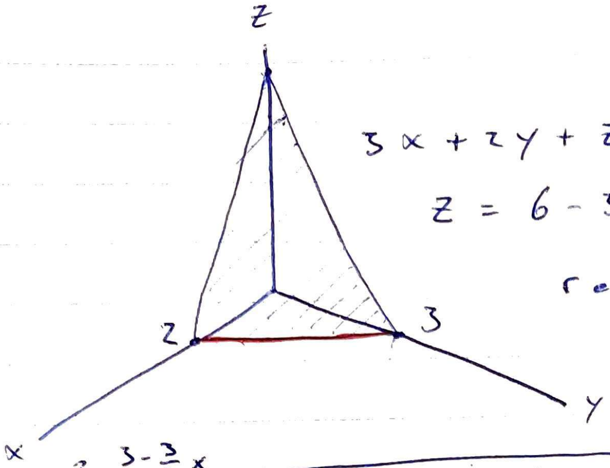
1)



$$3x + 2y + z = 6$$

$$z = 6 - 3x - 2y$$

red line : $3x + 2y = 6$

$$\Longrightarrow y = 3 - \frac{3}{2}x$$

$$A = \int_0^2 \int_0^{3-\frac{3}{2}x} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dy\, dx$$

$$= \int_0^2 \int_0^{3-\frac{3}{2}x} \sqrt{1 + (-3)^2 + (-2)^2}\, dy\, dx$$

$$= \int_0^2 \int_0^{3-\frac{3}{2}x} \sqrt{14}\, dy\, dx$$

$$= \int_0^2 3\sqrt{14} - \frac{3}{2}\sqrt{14}\, x\, dx$$

$$= \left[3\sqrt{14}\, x - \frac{3}{4}\sqrt{14}\, x^2\right]_0^2 = 3\sqrt{14}$$

2) $\iiint_E \sqrt{x^2+y^2}\, dV =$

E lies inside cylinder $x^2+y^2=16$ (radius 4) and between $z=-5$ and $z=4$

$$\int_0^{2\pi} \int_0^4 \int_{-5}^4 r^2\, dz\, dr\, d\theta =$$

$$\int_0^{2\pi} \int_0^4 9r^2\, dr\, d\theta =$$

$$\int_0^{2\pi} \left[3r^3\right]_0^4 d\theta =$$

$$\int_0^{2\pi} 192\, d\theta = 384\pi$$

3) $\iiint_E z \, dV =$

$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho\cos\phi\,\rho^2\sin\phi\,d\rho\,d\theta\,d\phi =$

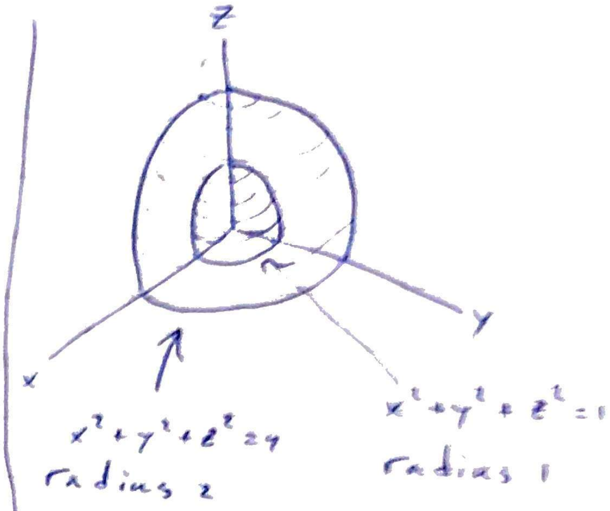
$\int_0^{\pi/2}\int_0^{\pi/2} \cos\phi\sin\phi\left[\frac{\rho^4}{4}\right]_1^2 d\theta\,d\phi =$

$\int_0^{\pi/2}\int_0^{\pi/2} \frac{15}{4}\cos\phi\sin\phi\,d\theta\,d\phi =$

$\int_0^{\pi/2} \frac{15\pi}{8}\cos\phi\sin\phi\,d\phi =$

$\int_0^{\pi/2} \frac{15\pi}{16}\sin 2\phi\,d\phi =$

$\frac{15\pi}{32}\left[-\cos 2\phi\right]_0^{\frac{\pi}{2}} = \frac{15\pi}{16}$



$\sin 2\phi = 2\sin\phi\cos\phi$

4) $x = uv, \quad y = vw, \quad z = uw$

$$\det\begin{pmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{pmatrix} = \det\begin{pmatrix} v & u & 0 \\ 0 & w & v \\ w & 0 & u \end{pmatrix}$$

$= v \cdot wu - u \cdot -wv = 2uvw$

5) $\iint_R x^2 \, dx\, dy =$

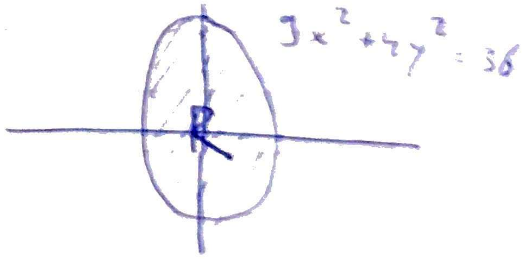
$\iint_S 4u^2 \cdot 6 \, du\, dv =$

$\int_0^{2\pi}\int_0^1 24 r^3 \cos^2\theta \, dr\, d\theta =$

$\int_0^{2\pi} \cos^2\theta\left[6r^4\right]_0^1 d\theta =$

$6\int_0^{2\pi}\cos^2\theta\,d\theta = 3\int_0^{2\pi} 1+\cos 2\theta\,d\theta$

$= 3\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{2\pi} = 6\pi$

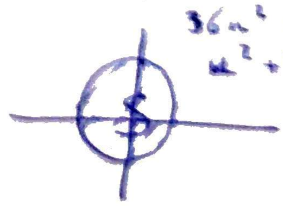


$\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = 6$

$\left\{ \begin{array}{l} x = 2u \\ y = 3v \end{array} \right.$

$36u^2 + 36v^2 = 36$

$u^2 + v^2 = 1$



6i) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$\nabla f(x,y,z) = \left(\frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2x \,,\, \frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2y \,,\, \frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2z\right)$$

$$= \frac{1}{\sqrt{x^2+y^2+z^2}}\,(x, y, z)$$

ii) $f(x,y) = \ln(x+2y)$

$$\nabla f(x,y) = \left(\frac{1}{x+2y} \,,\, \frac{2}{x+2y}\right) = \frac{1}{x+2y}\,(1, 2)$$

7i) $\int_C y \sin z \, ds =$ $\qquad C: t \mapsto (\cos t, \sin t, t)$, $t \in [0, 2\pi]$

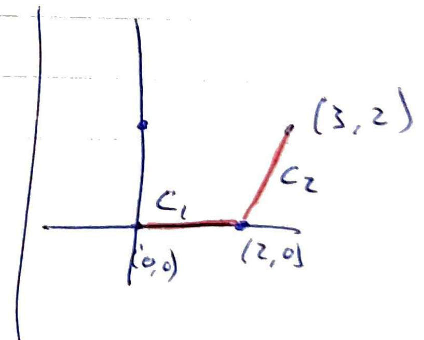
$$\int_0^{2\pi} \sin^2 t \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{2} \sin^2 t \, dt$$

$$= \frac{\sqrt{2}}{2} \int_0^{2\pi} 1 - \cos 2t \, dt \qquad \sin^2 t = \frac{1-\cos 2t}{2}$$

$$= \frac{\sqrt{2}}{2}\left[t - \frac{1}{2}\sin 2t\right]_0^{2\pi} = \pi\sqrt{2}$$

ii) $\int_C xy\, dx + (x-y)\, dy =$



$$\underbrace{\int_{C_1} xy\, dx}_{\substack{=0 \\ \text{because } y=0}} + \int_{C_2} xy\, dx + \underbrace{\int_{C_1} (x-y)\, dy}_{\substack{=0 \text{ because} \\ \frac{\partial y}{\partial t}=0}} + \int_{C_2} (x-y)\, dy =$$

$$\int_0^1 (4t + 2t^2)\frac{\partial x}{\partial t}\, dt + \int_0^1 (2-t)\frac{\partial y}{\partial t}\, dt =$$

$$\int_0^1 4t + 2t^2\, dt + \int_0^1 4 - 2t\, dt =$$

$$\left[2t^2 + \frac{2}{3}t^3\right]_0^1 + \left[4t - t^2\right]_0^1 = \frac{17}{3}$$

$C_1: t \mapsto (t, 0)$, $t \in [0, 2]$

$C_2: t \mapsto (2+t, 2t)$, $t \in [0, 1]$