Exercise Set 8

(1) Find the surface area of the part of the plane $3x+2y+z=6$ that lies in the first octant.

(2) Use cylindrical coordinates to evaluate the integral

$$\int\int\int_E \sqrt{x^2+y^2}\,dV$$

where E is the solid that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-5$ and $z=4$.

(3) Use spherical coordinates to evaluate the integral

$$\int\int\int_E z\,dV$$

where E is the solid that lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant.

(4) Compute the Jacobian determinant of the transformation $x=uv$, $y=vw$, $z=uw$.

(5) Use the transformation $x=2u$, $y=3v$ to evaluate $\int\int_R x^2\,dA$ where R is the region bounded by the ellipse $9x^2+4y^2=36$.

(6) Compute the gradient vector fields of the following functions:

(i) $f(x,\,y,\,z)=\sqrt{x^2+y^2+z^2}$
(ii) $f(x,\,y,)=\ln(x+2y)$

(7) Compute the following line integrals, where C is the given curve:

(i) $\int_C y\sin z\,ds$, C is the circular helix given by $t\mapsto(\cos t,\,\sin t,\,t)$ for $t\in[0,\,2\pi]$.
(ii) $\int_C xy\,dx+(x-y)\,dy$ where C is the line segment from $(0,\,0)$ to $(2,\,0)$ followed by the line segment from $(2,\,0)$ to $(3,\,2)$.