1) $A = \iint_R \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$

$R = [0,5] \times [1,4]$

$z = 2 + 3x + 4y$

$$= \int_1^4 \int_0^5 \sqrt{1 + 3^2 + 4^2}\, dx\, dy$$

$$= \int_1^4 \int_0^5 \sqrt{26}\, dx\, dy = 15\sqrt{26}.$$

2) $A = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$

$D = \{(x,y) : x^2 + y^2 \leq 1\}$

$z = xy$

$$= \int_0^{2\pi} \int_0^1 \sqrt{1 + y^2 + x^2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^1 \sqrt{1 + r^2}\, r\, dr\, d\theta$$

$u = 1 + r^2$

$du = 2r\, dr$

$$= \int_0^{2\pi} \int_1^2 \frac{1}{2}\sqrt{u}\, du\, d\theta$$

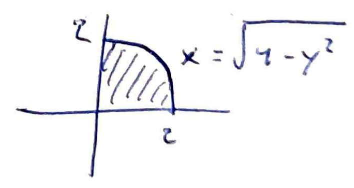
$$= \int_0^{2\pi} \left[\frac{1}{3} u^{\frac{3}{2}}\right]_{u=1}^{u=2} d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}\sqrt{2} - \frac{1}{3}\, d\theta = \frac{2\pi}{3}(2\sqrt{2} - 1).$$

3) $\iiint_E 2x\, dV$

$E = \{(x,y,z) : y \in [0,2]$, $x \in [0, \sqrt{4-y^2}]$, $z \in [x, 2x]\}$

$$= \int_0^2 \int_0^{\sqrt{4-y^2}} \int_x^{2x} 2x\, dz\, dx\, dy$$



$$= \int_0^2 \int_0^{\sqrt{4-y^2}} 2x^2\, dx\, dy$$

$$= \int_0^{\pi/2} \int_0^2 2r^3 \cos^2\theta\, dr\, d\theta$$

$$= \int_0^{\pi/2} \left[\frac{1}{2} r^4 \cos^2\theta\right]_{r=0}^{r=2} d\theta$$

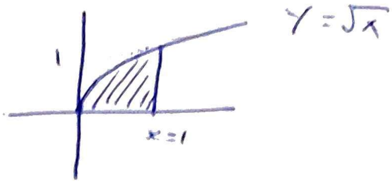
$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$$= \int_0^{\pi/2} 8\cos^2\theta\, d\theta$$

$$= 4\int_0^{\pi/2} 1 + \cos 2\theta\, d\theta = 4\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = 2\pi.$$

4 $$\iiint_E 6xy\,dV$$

$0 \le z \le 1 + x + y$



$$= \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy\,dz\,dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} 6xy + 6x^2y + 6xy^2\,dy\,dx$$

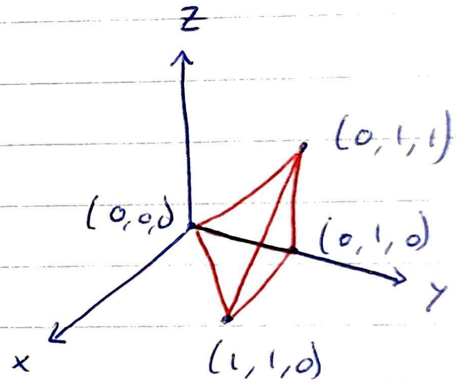
$$= \int_0^1 \left[3xy^2 + 3x^2y^2 + 2xy^3\right]_{y=0}^{y=\sqrt{x}}\,dx$$

$$= \int_0^1 3x^2 + 3x^3 + 2x^{\frac{5}{2}}\,dx$$

$$= \left[x^3 + \frac{3}{4}x^4 + \frac{4}{7}x^{\frac{7}{2}}\right]_{x=0}^{x=1}$$

$$= \frac{65}{28}$$

5) $$\iiint_E xz\,dV$$



$$= \int_0^1 \int_0^y \int_0^{y-x} xz\,dz\,dx\,dy$$

$$= \int_0^1 \int_0^y \left[\frac{1}{2}xz^2\right]_{z=0}^{z=y-x}\,dx\,dy$$

$$= \int_0^1 \int_0^y \frac{1}{2}x^3 - x^2y + \frac{1}{2}xy^2\,dx\,dy$$

$$= \int_0^1 \left[\frac{1}{8}x^4 - \frac{1}{3}x^3y + \frac{1}{4}x^2y^2\right]_{x=0}^{x=y}\,dy$$

$$= \int_0^1 \frac{1}{8}y^4 - \frac{1}{3}y^4 + \frac{1}{4}y^4\,dy$$

$$= \int_0^1 \frac{1}{24}y^4\,dy$$

$$= \left[\frac{1}{120}y^5\right]_0^1 = \frac{1}{120}$$

Bottom of tetrahedron:

$0 \le y \le 1, \quad 0 \le x \le y$

Top of tetrahedron:

$z = y - x$

$\Longrightarrow$ Inside tetrahedron:

$0 \le z \le y - x$

6) $\iiint_E x^2 \, dV$

$= \int_0^{2\pi} \int_0^1 \int_0^{2r} r^3 \cos^2\theta \, dz \, dr \, d\theta$

$= \int_0^{2\pi} \int_0^1 2r^4 \cos^2\theta \, dr \, d\theta$

$= \int_0^{2\pi} \left[\frac{2}{5} r^5 \cos^2\theta \right]_{r=0}^{r=1} d\theta$

$= \int_0^{2\pi} \frac{2}{5} \cos^2\theta \, d\theta$

$= \frac{1}{5} \int_0^{2\pi} 1 + \cos 2\theta \, d\theta$

$= \frac{1}{5} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{2\pi} = \frac{2\pi}{5}$

$E: \quad x^2 + y^2 \leq 1$

$z \geq 0$

$z^2 \leq 4x^2 + 4y^2$

In cylindrical coordinates:

$r^2 \leq 1 \implies r \leq 1$

$z^2 \leq 4r^2 \longrightarrow z \leq 2r$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

7) $\iiint_B x^2 + y^2 + z^2 \, dV$ $\qquad B = \{(x,y,z) : x^2 + y^2 + z^2 \leq 1\}$

$= \int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho^2 \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$= \int_0^{\pi} \int_0^{2\pi} \left[\frac{1}{5} \rho^5 \sin\phi \right]_{\rho=0}^{\rho=1} d\theta \, d\phi$

$= \int_0^{\pi} \int_0^{2\pi} \frac{1}{5} \sin\phi \, d\theta \, d\phi$

$= \int_0^{\pi} \frac{2\pi}{5} \sin\phi \, d\phi$

$= \frac{2\pi}{5} \left[-\cos\phi \right]_0^{\pi}$

$= \frac{4\pi}{5}$