1 i) $\int_1^3 \int_0^1 (1 + 4xy)\, dx\, dy =$

$\int_1^3 \left[ x + 2x^2 y \right]_{x=0}^{x=1} dy =$

$\int_1^3 1 + 2y\, dy =$

$\left[ y + y^2 \right]_{y=1}^{y=3} = 12 - 2 = 10.$

ii) $\int_1^4 \int_1^2 \left( \frac{x}{y} + \frac{y}{x} \right) dx\, dy =$

$\int_1^4 \left[ \frac{x^2}{2y} + y \log x \right]_{x=1}^{x=2} dy =$

$\int_1^4 \frac{3}{2y} + y \log 2\, dy =$

$\left[ \frac{3}{2} \log y + \frac{y^2}{2} \log 2 \right]_{y=1}^{y=4} = \frac{3}{2} \log 4 + \frac{15}{2} \log 2$

$= \frac{21}{2} \log 2$

2 i) $\iint_R \frac{xy^2}{x^2+1}\, dA$, where $R = [0,1] \times [-3,3]$

$= \int_0^1 \int_{-3}^3 \frac{xy^2}{x^2+1}\, dy\, dx$

$= \int_0^1 \frac{x}{x^2+1} \left[ \frac{y^3}{3} \right]_{y=-3}^{y=3} dx$

$= \int_0^1 \frac{18x}{x^2+1}\, dx$ $\qquad u = x^2+1$

$du = 2x\, dx$

$= \int_1^2 \frac{9}{u}\, du$

$= \left[ 9 \log u \right]_1^2 = 9 \log 2$

2ii) $\iint_R x\sin(x+y)\, dA$ , $R = [0, \frac{\pi}{6}] \times [0, \frac{\pi}{3}]$

$$= \int_0^{\pi/3}\int_0^{\pi/6} x\sin(x+y)\, dx\, dy$$

$$= \int_0^{\pi/3} \Big[-x\cos(x+y) + \sin(x+y)\Big]_{x=0}^{x=\frac{\pi}{6}} dy$$

$$= \int_0^{\pi/3} -\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right) - \sin(y)\, dy$$

$$= \left[-\frac{\pi}{6}\sin\left(\frac{\pi}{6}+y\right) - \cos\left(\frac{\pi}{6}+y\right) + \cos(y)\right]_0^{\pi/3}$$

$$= -\frac{\pi}{6} - 0 + \frac{1}{2} - \left(-\frac{\pi}{6}\cdot\frac{1}{2} - \frac{1}{2}\sqrt{3} + 1\right)$$

$$= \frac{-6 + 6\sqrt{3} - \pi}{12}$$

3) $\int_{-2}^{3}\int_0^1 z\, dx\, dy$ $\qquad 3x + 2y + z = 12$

$\Longrightarrow z = 12 - 3x - 2y$

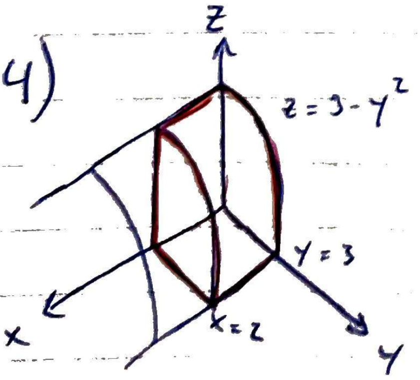
$$= \int_{-2}^{3}\int_0^1 12 - 3x - 2y\, dx\, dy$$

$$= \int_{-2}^{3} \left[12x - \frac{3}{2}x^2 - 2xy\right]_{x=0}^{x=1} dy$$

$$= \int_{-2}^{3} 12 - \frac{3}{2} - 2y\, dy = \int_{-2}^{3} \frac{21}{2} - 2y\, dy$$

$$= \left[\frac{21}{2}y - y^2\right]_{-2}^{3} = \frac{95}{2}$$

4)

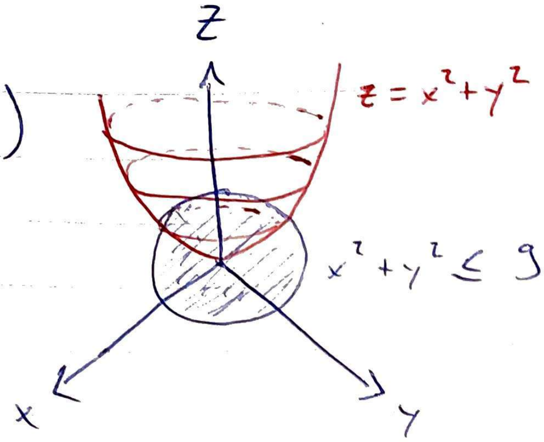



$$\int_0^3\int_0^2 9 - y^2\, dx\, dy =$$

$$\int_0^3 \left[9x - xy^2\right]_{x=0}^{x=2} dy =$$

$$\int_0^3 18 - 2y^2\, dy =$$

$$\left[18y - \frac{2}{3}y^3\right]_{y=0}^{3} = 36$$

5) 


$$\int_0^{2\pi}\int_0^3 z\, r\, dr\, d\theta =$$

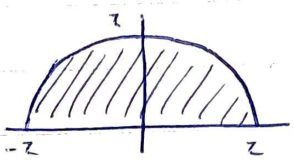
$$\int_0^{2\pi}\int_0^3 r^3\, dr\, d\theta =$$

$$\int_0^{2\pi} \left[\frac{1}{4}r^4\right]_{r=0}^{3} d\theta =$$

$$\int_0^{2\pi} \frac{81}{4}\, d\theta = \frac{81\pi}{2}$$

$$Z = x^2+y^2 = r^2$$

6) $$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2\, dx\, dy =$$




$$\int_0^{\pi}\int_0^2 r^2\cos^2\theta\, r^2\sin^2\theta\, r\, dr\, d\theta =$$

$$\int_0^{\pi} \left[\frac{r^6}{6}\cos^2\theta\, \sin^2\theta\right]_{r=0}^{r=2} d\theta =$$

$$\frac{64}{6}\int_0^{\pi} \cos^2\theta\, \sin^2\theta\, d\theta =$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$
$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\frac{64}{6}\int_0^{\pi} \frac{(1+\cos(2\theta))(1-\cos(2\theta))}{4}\, d\theta =$$

$$\frac{16}{6}\int_0^{\pi} 1 - \cos^2 2\theta\, d\theta =$$

$$\frac{16}{6}\int_0^{\pi} 1 - \frac{1+\cos(4\theta)}{2}\, d\theta =$$

$$\frac{16}{6}\left[\frac{1}{2}\theta - \frac{1}{8}\sin(4\theta)\right]_0^{\pi} = \frac{4\pi}{3}$$

7) $$\int_0^1\int_0^{x^2} x\cos y\, dy\, dx =$$

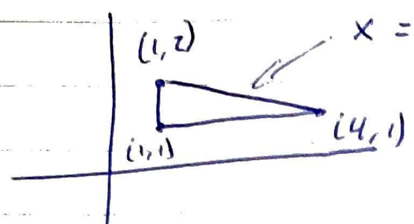
$$\int_0^1 [x\sin y]_{y=0}^{y=x^2}\, dx =$$

$$\int_0^1 x\sin x^2\, dx =$$

$$u = x^2$$
$$du = 2x\, dx$$

$$\int_0^1 \frac{1}{2}\sin u\, du = \left[-\frac{1}{2}\cos u\right]_0^1 = -\frac{1}{2}\cos 1 + \frac{1}{2} = \sin^2\frac{1}{2}$$

8)



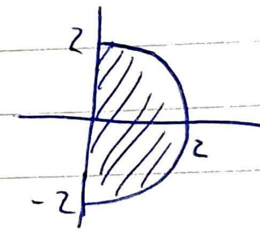

$$\int_1^2 \int_1^{-3y+7} xy \, dx \, dy =$$

$$\int_1^2 \left[\frac{1}{2}x^2 y\right]_{x=1}^{-3y+7} dy =$$

$$\int_1^2 \frac{1}{2}(9y^3 - 42y^2 + 48y) \, dy =$$

$$\left[\frac{9}{8}y^4 - 7y^3 + 12y^2\right]_{y=1}^{2} = \frac{31}{8}$$

9) $\iint_D e^{-x^2-y^2} \, dA =$




$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 e^{-r^2} r \, dr \, d\theta =$$

$u = r^2$

$du = 2r \, dr$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^4 \frac{1}{2} e^{-u} \, du \, d\theta =$$

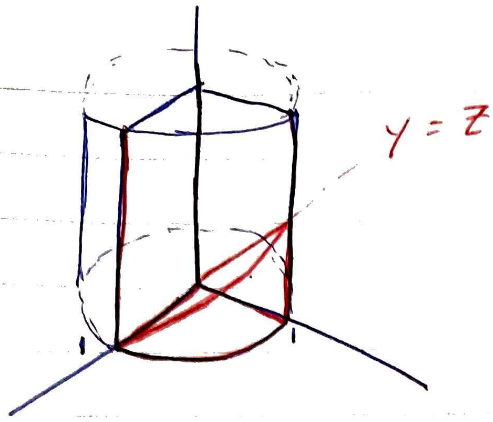
$$\int_{-\frac{\pi}{2}}^{\pi/2} \left[-\frac{1}{2}e^{-u}\right]_{u=0}^{u=4} d\theta =$$

$$\int_{-\pi/2}^{\pi/2} -\frac{1}{2}e^{-4} + \frac{1}{2} \, d\theta =$$

$$\left[\left(-\frac{1}{2}e^{-4} + \frac{1}{2}\right)\theta\right]_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$\frac{\pi}{2}\left(-e^{-4} + 1\right) = \frac{\pi}{2e^4}\left(e^4 - 1\right)$$

10




$$\int_0^{\pi/2}\int_0^1 z\, r\, dr\, d\theta =$$

$$\int_0^{\pi/2}\int_0^1 r^2 \sin\theta\, dr\, d\theta =$$

$$\int_0^{\pi/2} \left[\frac{1}{3} r^3 \sin\theta\right]_{r=0}^{r=1} d\theta =$$

$$\int_0^{\pi/2} \frac{1}{3}\sin\theta\, d\theta =$$

$$\left[-\frac{1}{3}\cos\theta\right]_{\theta=0}^{\theta=\frac{\pi}{2}} = \frac{1}{3}$$