Exercise Set 6

MA2071

(1) Calculate the iterated integrals:

(i) $\displaystyle\int_1^3 \int_0^1 (1+4xy)\,dx\,dy.$

(ii) $\displaystyle\int_1^4 \int_1^2 (\tfrac{x}{y}+\tfrac{y}{x})\,dx\,dy.$

(2) Calculate the double integrals:

(i) $\displaystyle\int\int_R \frac{xy^2}{x^2+1}\,dA$, where $R=[0,\,1]\times[-3,\,3]$.

(ii) $\displaystyle\int\int_R x\sin(x+y)\,dA$, where $R=[0,\,\tfrac{\pi}{6}]\times[0,\,\tfrac{\pi}{3}]$.

(3) Compute the volume of the solid that lies under the plane $3x+2y+z=12$ and above the rectangle $[0,\,1]\times[-2,\,3]$.

(4) Find the volume of the solid in the first octant bounded by the cylinder $z=9-y^2$ and the plane $x=2$.

(5) Find the volume of the solid under the paraboloid $z=x^2+y^2$ and above the disk $x^2+y^2\leq 9$.

(6) Evaluate $\displaystyle\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2\,y^2\,dx\,dy.$

(7) Compute $\displaystyle\int\int_D x\,\cos y\,dA$ where D is the region bounded by $y=0$, $y=x^2$, $x=0$ and $x=1$.

(8) Find the volume of the solid between the surface $z=xy$ and the triangle in the plane $z=0$ with vertices $(1,\,1)$, $(4,\,1)$ and $(1,\,2)$.

(9) Compute $\displaystyle\int\int_D e^{-x^2-y^2}\,dA$ where D is the region bounded by the semicircle $x=\sqrt{4-y^2}$ and the y-axis.

(10) Find the volume of the solid bounded by the cylinder $x^2+y^2=1$ and the planes $y=z$, $x=0$, $z=0$ in the first octant.