

Exercise Set 4

MA2071

(1) Find the directional derivative of f at $P = [x_0, y_0, z_0]$ in the direction v .

(i) $f(x, y) = 1 + 2x\sqrt{y}$, $P = [3, 4]$, $v = [4, -3]$

(ii) $f(x, y, z) = x/(y + z)$, $P = [4, 1, 1]$, $v = [1, 2, 3]$

(2) Find the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at $P = [2, 1, 3]$ in the direction of the origin.

(3) Find the maximal and minimal rates of change of f at the given point P , and determine the direction in which this occurs.

(i) $f(x, y, z) = x^2y^3z^4$ at $P = [1, 1, 1]$.

(ii) $f(x, y, z) = \ln(xy^2z^3)$ at $P = [1, -2, -3]$.

(4) Find equations of the tangent planes of the given surfaces at the specified points.

(i) $x^2 + 2y^2 + 3z^2 = 21$ at $P = [4, -1, 1]$

(ii) $x - z = 4 \arctan(yz)$ at $P = [1 + \pi, 1, 1]$.

(5) Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ at which the tangent plane is parallel to the plane $3x - y + 3z = 1$.