Exercise Set 3 MA2071

(1) Find the equation of the tangent plane of the given surface at the specified point.

(i) $z = \sqrt{4 - x^2 - 2y^2}$ at $p = (1, -1, 1)$
(ii) $z = y \log x$ at $p = (1, 4, 0)$

(2) Find the differentials of the following functions

(i) $z = x^3 \log(y^2)$
(ii) $w = \log \sqrt{x^2 + y^2 + z^2}$

(3) Use the chain rule to find dz/dt and dw/dt for

(i) $z = \sin x \cos y$ where $x = \pi t,\ y = \sqrt{t}$
(ii) $w = xy + yz^2$ where $x = e^t,\ y = e^t \sin t,\ z = e^t \cos t$

(4) Use the chain rule to find the indicated partial derivatives

(i) $z = x^2 + xy^3$ where $x = uv^2 + w^3,\ y = u + ve^w$.
Find $\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}, \dfrac{\partial z}{\partial w}$ when $u = 2,\ v = 1,\ w = 0$

(ii) $R = \log(u^2 + v^2 + w^2)$ where $u = x + 2y,\ v = 2x - y,\ w = 2xy$.
Find $\dfrac{\partial R}{\partial x}, \dfrac{\partial R}{\partial y}$ when $x = y = 1$

(5) Let $z = f(x, y)$.

(i) Suppose $x = r\cos\theta,\ y = r\sin\theta$.
(a) Show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

(b) Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}$$

(ii) Suppose $x = s + t,\ y = s - t$. Show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s}\frac{\partial z}{\partial t}$$