1) $dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$

$= \sqrt{2 + 4y^2}\, dx\, dy$

$\Rightarrow \iint_S y\, dS = \int_0^2 \int_0^1 y \sqrt{2 + 4y^2}\, dx\, dy$

$= \int_0^2 y\sqrt{2+4y^2}\, dy$ $\qquad u = 2 + 4y^2$

$\qquad du = 8y\, dy$

$= \int_2^{18} \frac{1}{8}\sqrt{u}\, du$

$= \left[\frac{1}{12} u^{\frac{3}{2}}\right]_2^{18} = \frac{13\sqrt{2}}{3}$

2) $dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx dy = \sqrt{14}\, dx\, dy$

$\Rightarrow \iint_S x^2 y z\, dS =$

$\int_0^2 \int_0^3 x^2 y (1 + 2x + 3y) \sqrt{14}\, dx\, dy =$

$\sqrt{14} \int_0^2 \int_0^3 x^2 y + 2x^3 y + 3x^2 y^2\, dx\, dy =$

$\sqrt{14} \int_0^2 \left[\frac{1}{3} x^3 y + \frac{1}{2} x^4 y + x^3 y^2\right]_{x=0}^{x=3} dy =$

$\sqrt{14} \int_0^2 \frac{99}{2} y + 27y^2\, dy =$

$\sqrt{14}\left[\frac{99}{4} y^2 + 9y^3\right]_{y=0}^{y=2} = 171\sqrt{14}$

3) $dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx dy = \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy$

S is given by $f(x,y,z) = z - 4 + x^2 + y^2 = 0$

$\Rightarrow \nabla f = (2x, 2y, 1)$ is a normal vector

$\Rightarrow \underline{n} = \frac{\nabla f}{\|\nabla f\|} = \frac{(2x, 2y, 1)}{\sqrt{4x^2 + 4y^2 + 1}}$

$$\iint_S F\cdot \underline{n}\, dS = \int_0^1\int_0^1 (xy,\ yz,\ zx)\cdot \frac{(2x,\ 2y,\ 1)}{\sqrt{4x^2+4y^2+1}}\sqrt{1+4x^2+4y^2}\ dx\,dy$$

$$= \int_0^1\int_0^1 2x^2y + 2y^2z + zx\ \ dx\,dy$$

$$= \int_0^1\int_0^1 2x^2y + 2y^2(4-x^2-y^2) + (4-x^2-y^2)x\ dx\,dy$$

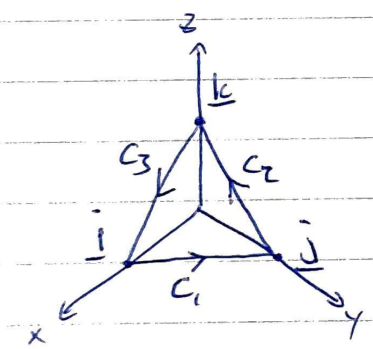
$$= \int_0^1\int_0^1 2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2\ dx\,dy$$

$$= \int_0^1 \left[\tfrac{2}{3}x^3y + 8xy^2 - \tfrac{2}{3}x^3y^2 - 2xy^4 + 2x^2 - \tfrac{1}{4}x^4 - \tfrac{1}{2}x^2y^2\right]_{x=0}^{x=1} dy$$

$$= \int_0^1 \tfrac{7}{4} + \tfrac{2}{3}y + \tfrac{41}{6}y^2 - 2y^4\ dy$$

$$= \left[\tfrac{7}{4}y + \tfrac{1}{3}y^2 + \tfrac{41}{18}y^3 - \tfrac{2}{5}y^5\right]_0^1 = \frac{713}{180}$$

4)



$$\left.\begin{aligned} C_1 &: (1-t,\ t,\ 0)\\ C_2 &: (0,\ 1-t,\ t)\\ C_3 &: (t,\ 0,\ 1-t)\end{aligned}\right\}\ t\in[0,1]$$

$$\int_{C_1} F\,ds = \int_0^1 F(1-t,\ t,\ 0)\cdot(-1,\ 1,\ 0)\ dt$$

$$= \int_0^1 (1-t+t^2,\ t,\ (1-t)^2)\cdot(-1,\ 1,\ 0)\ dt$$

$$= \int_0^1 -1 + 2t - t^2\ dt$$

$$= \left[-t + t^2 - \tfrac{1}{3}t^3\right]_0^1 = -\tfrac{1}{3}.$$

Similarly, $\int_{C_2} F\,ds = \int_{C_3} F\,ds = -\frac{1}{3}$

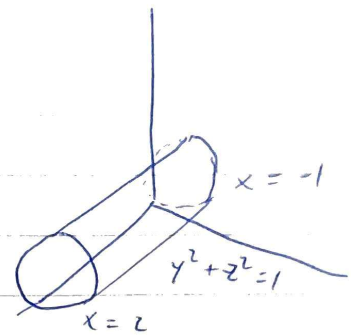
$$\Longrightarrow \int_C F\,ds = \int_{C_1} F\,ds + \int_{C_2} F\,ds + \int_{C_3} F\,ds = -1.$$

5) $\text{curl } F = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$

$$= (x - x, y - y, z - z) = 0$$

$$\Longrightarrow \iint_S \text{curl } F \cdot \underline{n} \, dS = 0.$$

6) $\iint_S F \cdot \underline{n} \, dS =$



$\iiint_V \text{div } F \, dV =$

$\iiint_V 3y^2 + 3z^2 \, dV$

Use the coordinate transformation

$x = x, \quad y = r\cos\theta, \quad z = r\sin\theta$

$dV = r \, dx \, dr \, d\theta$

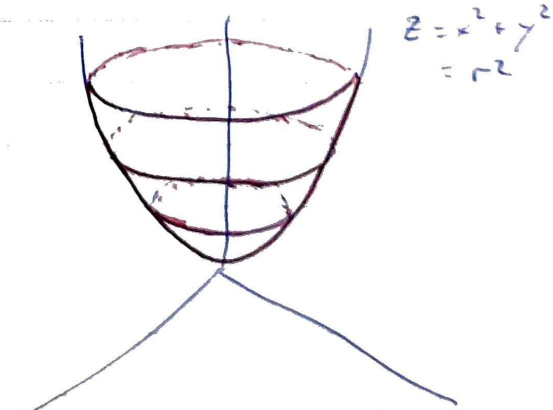
$$= \int_0^{2\pi} \int_0^1 \int_{-1}^2 3r^3 \, dx \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 9r^3 \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[\frac{9}{4} r^4 \right]_{r=0}^{r=1} d\theta$$

$$= \int_0^{2\pi} \frac{9}{4} \, d\theta = \frac{9}{2}\pi$$

7) $\iint_S F \cdot \underline{n} \, dS =$



$\iiint_V \text{div } F \, dV =$

$\iiint_V y^2 + x^2 \, dV =$

$$\int_0^{2\pi} \int_0^2 \int_{r^2}^4 r^3 \, dz \, dr \, d\theta$$

Intersection of
$z = x^2 + y^2$ and $z = 4$:
$x^2 + y^2 = 4$

$$= \int_0^{2\pi} \int_0^{2} 4r^3 - r^5 \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[r^4 - \frac{1}{6} r^6 \right]_{r=0}^{r=2} d\theta$$

$$= \int_0^{2\pi} \frac{16}{3} \, d\theta = \frac{32}{3} \pi$$