Exercise Set 1 MA2071

For $\mathbb{R}^3$ it is convenient to introduce notation for the standard basis

$$\mathbb{i} = \begin{bmatrix}1\\0\\0\end{bmatrix}, \quad \mathbb{j} = \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \mathbb{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}.$$

(1) (a) At what point does the curve $t \mapsto t\,\mathbb{i}+(2t-t^2)\,\mathbb{k}$ intersect the paraboloid $z = x^2+y^2$?
(b) Find centre and radius of the sphere $x^2+y^2+z^2-8x+2y+6z+1=0$.

(2) (a) Find a parametrization of the curve of intersection of the paraboloid $z = 4x^2+y^2$ and the parabolic cylinder $y = x^2$.
(b) Find a parametrization of the tangent line to the curve $F(t) = (t^2-1)\mathbb{i}+(t^2+1)\mathbb{j}+(t+1)\mathbb{k}$ at the point $-\mathbb{i}+\mathbb{j}+\mathbb{k}$.

(3) Suppose the trajectories of two moving particles are given by the curves $\gamma_1(t) = t^2\mathbb{i} + (7t-12)\mathbb{j}+t^2\mathbb{k}$ and $\gamma_2(t) = (4t-3)\mathbb{i}+t^2\mathbb{j}+(5t-6)\mathbb{k}$ for $t \geq 0$. Will the particles collide?

(4) (a) Find the point of intersection of the line $F(t) = (2+3t)\,\mathbb{i} - 4t\,\mathbb{j} + (5+t)\,\mathbb{k}$ and the plane $4x+5y-2z = 18$.
(b) Find the equation of the plane that contains the three points $\mathbb{i}$, $2\mathbb{j}$ and $3\mathbb{k}$.
(c) Show that the planes $x+y-z = 1$ and $2x-3y+4z = 5$ are neither parallel nor perpendicular. Compute the angle between these planes.
(d) Find the equation of the plane through the point $2\mathbb{i}+\mathbb{j}$ parallel to the plane $x+4y-3z = 1$.

(5) (a) The position of a moving particle at time $t \geq 0$ is given by the map $F(t) = t^2\mathbb{i}+5t\mathbb{j}+(t^2-16t)\mathbb{k}$. At what time is the speed the least?
(b) Show that if a particle moves at constant speed, then velocity and acceleration vectors are perpendicular.

(6) Find a parametrization of the lower half of the ellipsoid with half-axes $(a, b, c) = (2, 1, 3)$.

(7) Find a parametrization of the surface of revolution obtained by rotating the curve $x = 4y^2-y^4$, $-2 \leq y \leq 2$ about the y-axis. Sketch the resulting surface.