KNOT THEORY AND CLUSTER ALGEBRAS

RALF SCHIFFLER (CONNECTICUT)

Abstract. We associate a cluster algebra to every knot or link in such a way that the Alexander polynomial of the knot is recovered from certain cluster variables under specialization. More precisely, given a link diagram K, we define a quiver Q whose vertices correspond to the segments of K and whose arrows go clockwise around the crossing points of K. For every segment i of K, we construct an indecomposable quiver representation $T(i)$ by defining its vector spaces and linear maps. In the cluster algebra, we construct corresponding cluster variables $x(i)$ by specifying an explicit mutation sequence. In particular, the collection of all $x(i)$ forms a cluster in the cluster algebra. Each $x(i)$ (and the F-polynomial of each $T(i)$) specialize to the Alexander polynomial of K, under the same specialization.

NAHM SUMS FROM CLUSTER TRANSFORMATIONS

YUMA MIZUNO (CORK)

Abstract. In the 1990s, it was actively studied in mathematical physics that the characters of certain conformal field theories can be expressed as q-hypergeometric sums, now often called Nahm sums. Nahm sums are also related to various areas of mathematics, including dilogarithm identities, Y-systems, combinatorics, Rogers-Ramanujan identities, invariants of 3-manifolds, and cluster algebras. In this talk, I will give an introductory overview focusing especially on their connection with cluster algebras. I will explain the construction of Nahm sums expected to be modular from cluster transformations of finite order.

MODULAR NAHM SUMS FROM CLUSTER ALGEBRAS

DMITRY NOSHCHENKO (DIAS)

Abstract. We will discuss the new surprising relationship between cluster algebras associated to triangulated surfaces and modular Nahm sums. Our motivation comes from physics – namely, the interface between four- and three-dimensional gauge theories with supersymmetry. On the one hand, the Nahm sums in question represent holomorphic blocks in three dimensions, and on the other reproduce characters of vertex operator algebras related to BPS quivers and their mutations. The talk is partially based on arXiv:2508.09729, as well as an ongoing work with V. Dotsenko, B. Feigin, P. Kucharski and W. Li.

HIGHER TEICHMÜLLER THEORY AND CLUSTER ALGEBRAS

TSUKASA ISHIBASHI (TOHOKU)

Abstract. For a compact surface with marked points, Fock and Goncharov introduced the moduli space of G-local systems on the surface equipped with decoration data at the marked points. This moduli space admits a distinguished atlas called a cluster K_2 structure, whose positive real part is a decorated version of higher Teichmüller space. The cluster structure provides a natural framework for quantizing the moduli space, and the resulting quantum theory is expected to be intimately related to the complex Chern–Simons theory. In this introductory talk, I will explain the geometry of this cluster structure and discuss the basic ideas behind its quantization. I will also describe its connections with skein algebras and Kashaev-type quantum invariants.

NONCOMMUTATIVE CLUSTER ALGEBRAS IN HIGHER TEICHMÜLLER THEORY

ZACK GREENBERG (LEIPZIG)

Abstract. The relationship between classic Teichmüller theory (representations of $\pi_1(S)$ into $\mathrm{SL}_2(\mathbb{R})$) and cluster algebras is well-studied. By reframing the cluster algebra in terms of "angles" at the marked points of the surface, we can define nice non-commutative generalizations of cluster algebras. This generalization is exactly the right object to study representations into nonsplit Lie groups G. When G has an additional positive structure, we parameterize the set of "Theta-positive" representations which generalize higher Teichmüller spaces.

SKEIN AND CLUSTER ALGEBRAS OF UNPUNCTURED SURFACES FOR RANK 2 SIMPLE LIE ALGEBRAS

WATARU YUASA (HIROSHIMA)

Abstract. This talk surveys recent developments in the relation between skein algebras and quantum cluster algebras for rank-two simple Lie algebras $\mathfrak{g} = \mathfrak{sl}_3$, $\mathfrak{sp}_4$, $\mathfrak{g}_2$. I will explain how $\mathfrak{g}$-webs on unpunctured marked surfaces are related to quantum cluster algebras associated with moduli spaces of decorated local systems on the surface. The main goal of the talk is to present the general picture, in particular, how one constructs embeddings of skein algebras into the corresponding quantum cluster algebras. I will also present explicit examples of $\mathfrak{g}$-webs corresponding to cluster variables.

TOPOLOGICAL DEFECTS AND QUANTUM CLUSTER COORDINATES

JENNIFER BROWN (EDINBURGH)

Abstract. Character varieties depend on a topological space and a reductive group, but some constructions most naturally involve assigning different groups to different parts of the space. This can be done using topological defects, which form interfaces between the different types of local systems parametrized by the character variety. We'll focus on how this physically inspired story plays out for quantum cluster coordinates, using skein theory. This talk is based on works in collaboration with Matthias Vancraeynest, David Jordan, and Juan Ramòn Gòmez Garcìa.

CLUSTERS AND LEGENDRIAN 3-BRAIDS

JAMES HUGHES (DUKE)

Abstract. An exact Lagrangian filling of a Legendrian link L is a Lagrangian surface in the standard symplectic 4-ball with boundary equal to L. Over the past decade, cluster-theoretic tools have significantly advanced the classification of these objects, with particular success achieved through the study of braid varieties, i.e., algebraic varieties that behave like a "moduli of fillings" for certain Legendrian braid closures. In this talk, I will discuss joint work in progress with Lenny Ng and Daping Weng in which we explore a generalization of braid varieties associated to any Lagrangian fillable Legendrian 3-braid and establish new phenomena, including examples with multiple irreducible components and others for which no cluster structure appears to exist.

ALTERNATING SNAKE MODULES AND CLUSTER ALGEBRAS

JINGMIN GUO (LEIPZIG)

Abstract. Hernandez and Leclerc discovered a connection between finite-dimensional representations of quantum affine algebras and cluster algebras by introducing a monoidal categorification of certain cluster algebras. They conjectured that cluster variables correspond to real prime simple modules and cluster monomials correspond to real simple modules. In this talk, I will first review the cluster algebra introduced by Hernandez and Leclerc. I will then introduce alternating snake modules, introduced by Matheus Brito and Vyjayanthi Chari, which generalize both snake modules and HL-modules. Finally, I will present our ongoing work on a class of real prime alternating snake modules. The main focus will be on constructing explicit mutation sequences, computing the corresponding cluster variables, and identifying the corresponding real prime simple modules. This talk is based on joint work with Matheus Brito, Vyjayanthi Chari, and Jian-Rong Li.

UNIVERSAL F-POLYNOMIALS AND FINITE-DIMENSIONAL ALGEBRAS

PIERRE-GUY PLAMONDON (VERSAILLES)

Abstract. Starting from an algebra of finite representation type, we define the universal F-polynomial of a module, and look at the variety defined by specializing all universal F-polynomials to 1. This generalizes varieties defined in Dynkin types by Arkani-Hamed, He and Lam, and is related to cluster theory and to classical objects in geometry. I will present the constructions of the objects and state some of the main results and open questions. This is a joint work with Arkani-Hamed, Frost, Salvatori and Thomas.