

CYCLIC QUANTUM TEICHMÜLLER THEORY

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ABSTRACT. Based on the pioneering ideas of Kashaev, we present a fully explicit construction of a finite-dimensional projective representation of the dotted Ptolemy groupoid when the quantum parameter q is a root of unity. We introduce the quantum intertwiner associated with a mapping class as a composite of cyclic quantum dilogarithm operators, whose trace defines a quantum invariant. This invariant coincides with the reduced quantum hyperbolic operator of Baseilhac–Benedetti, which constitute a “reduced” part of Kashaev’s link invariant. We further provide a geometric method to decompose the space of quantum states into irreducible modules of the Chekhov–Fock algebra. The reduced version of the quantum intertwiner conjecturally coincides with the Bonahon–Liu intertwiner, which is also conjectured to produce the 1-loop invariant. Altogether, this work offers a new perspective on the relationship among these quantum invariants.