CRANE–YETTER WITH BACKGROUND FIELDS: TWO APPROACHES

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Topological quantum field theories (TQFTs) are quantum field theories whose correlation functions are invariant under deformations of spacetime. A classic example is Chern–Simons: a 3d gauge theory, whose fields are principal G-bundles equipped with a connection, with classical solutions being the flat connections. Unlike QFTs, which have no universally agreed-upon mathematical definition, TQFTs were axiomatised by Atiyah as symmetric monoidal functors from the category of manifolds (space) and cobordisms (spacetime) to a chosen category of an algebraic flavour (capturing sate spaces and time evolution). This allows us to study aspects of QFT in a rigorous mathematical setting, where we can apply the tools developed to study topology and (higher) category theory.

In 1989, Witten showed that quantised Wilson loop observables in Chern–Simons recover the Jones polynomial of knots. Around the same time, early examples of functorial TQFTs were given: the 3d TQFT of Reshtikhin–Turaev, which uses the graphical calculus of a modular tensor category, and is constructed using a Morse-theoretic presentation of a manifold; and the 4d theory of Crane–Yetter, which is constructed using a triangulation of the manifold. These TQFTs have an explicit combinatorial description, and satisfy the cutting-and-gluing properties needed to define a functor from the category of cobordisms. In 1995, Baez and Dolan conjectured that fully-extended TQFTs (those which satisfy higher-codimension cutting-and-gluing, and can be defined as functors from the higher category of points, 1d-bordisms, 2d cobordisms with corners, and so on) should be in correspondence with fully-dualizable objects of the target category. An influential sketch of this so-called Cobordism Hypothesis was given by Lurie, but to date no widely-accepted proof has appeared. This result gives a powerful organising principle which predicts the existence of TQFTs, but specifying fully-extended target data does not generally allow us to construct manifold invariants in the same way as the non-extended, combinatorial approaches of Reshetikhin–Turaev/Crane–Yetter.

In this talk I will describe how the interplay between these two perspectives on TQFT can be productive. I will mention previous work in which I used the Cobordism Hypothesis to predict the existence of an upgraded version of Crane–Yetter for manifolds equipped with a flat connection, allowing us to couple to the moduli space of classical solutions in gauge theory. I will also describe ongoing work with Jordan, Haïoun and Van Dyke, on giving a combinatorial description of this predicted TQFT.