MARKOFF-LIKE SURFACES AND DIVISIBILITY OF ORBITS

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ABSTRACT. Markoff triples are deeply woven into the fabric of modern mathematics, originating with A.A. Markoff's work in the late 1870's while continuing to make appearances in number theory, hyperbolic geometry, cluster algebras, and beyond. A recent breakthrough of W.Y. Chen was that if we consider solutions to the Markoff equation $(x^2+y^2+z^2=xyz)$ over fields of prime order, then all non-trivial orbits of the solution set under the action of Vieta jumping have size divisible by p. In this talk we will discuss recent work on a family of Markoff-like surfaces with extra off-diagonal terms, and elaborating on an even more recent argument of D.E. Martin, show that for a typical surface in this family every non-trivial orbit has size divisible by p. We expect that there is just one orbit generically and for some special parameters, we will show that there are at least two or four orbits. For the exceptional cases when divisibility does not hold, we provide explicit constructions of orbits not divisible by p. This is based on joint work with M. de Courcy-Ireland (Stockholm University) and Y. Mizuno (UCC).