ASYMPTOTIC CHARACTERISATION OF SOLUTIONS OF PERTURBED CONTINUOUS-TIME DIFFERENTIAL SYSTEMS

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Abstract. A typical situation in the study of qualitative (especially asymptotic) properties of continuous-time dynamical systems (and indeed mathematical models), is the following. The underlying problem, without external forcing, exhibits a stability property, or more generally, has certain well-understood dynamical properties. The system is then subjected to an external shock, which is persistent, but can fade. A natural question therefore is: what are characterising conditions on the shock and underlying system, so that the dynamics of the underlying problem are preserved? An equally natural and related question is: if, after the shock, the solution possesses new properties, can we characterise what class of perturbations would generate these properties?

Usually, in dynamical systems, we consider that good progress has been made if sufficient conditions for the preservation of the underlying dynamics can be found. However, in this talk, we will summarise for various classes of differential system and perturbations when coinciding necessary and sufficient conditions can be found so that the unperturbed dynamics are preserved, or that solutions of the perturbed equations exhibit new properties. The equations studied include linear and nonlinear ordinary and stochastic differential equations, as well as differential equations with memory (such as delay-differential equations, or Volterra equations).

Remarkably, what turns out to be central in many cases is the behaviour of very simple perturbed scalar linear autonomous differential equations, revealing that certain moving averages of both deterministic and stochastic perturbations are often crucial. Even more remarkably, given the simplicity of these perturbed linear equations, is that a characterisation of when their solutions belong to certain important function spaces is not heretofore known. Finally, it transpires that differential systems seem to be more forgiving to shocks than integral or discrete systems, and we will comment on this too.

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