WHY DOES SUB-RIEMANNIAN GEOMETRY MATTER IN HARMONIC ANALYSIS?

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In wide areas of non-commutative Harmonic Analysis so-called sub-Laplacians play a prominent role. Such operators typically induce a sub-Riemannian geometry, for which the given operator forms the analogue of the Laplace-Beltrami operator. Sub-Riemannian geometry is an important and active branch of differential geometry, which dates back already to ideas of Carnot and Carathéodory.

It has turned out that these geometries play a crucial role for a deeper understanding of problems in Harmonic Analysis in such settings.

I shall explain these concepts in a very concrete and elementary way for the sub-Laplacian on the Heisenberg group $\mathbb{H}_n$, mostly for the case of the 3-dimensional Heisenberg group $\mathbb{H}_1$.

As a very striking example of this philosophy, I shall present a recent result obtained in joint work with L. Niedorf, A. Seeger and B. Stovall, in which we prove the first really interesting boundedness results for Bochner–Riesz means associated with the spectral decomposition of the sub-Laplacian on the Heisenberg group.

Earlier attempts to prove such results by means of Fourier restriction type estimates in an analogous way as in Euclidean Harmonic Analysis had failed, since the analogous spectral restriction estimates do not hold on the Heisenberg group.