A BRIEF HISTORY OF TRIANGULATIONS

CHRISTOPHER BISHOP (STONY BROOK)

Abstract. A fundamental problem of mathematics (and of science in general) is to decompose a complex object into simpler pieces, e.g., integers into products of primes, or functions into Fourier waveforms. One of the oldest such problems is to cut a general polygon into triangles (the simplest type of polygon). In this lecture, I will survey about 200 years of progress on triangulations, starting with the Wallace–Bolyai–Gerwien theorem; that any two equal area polygons can be dissected into identical sets of triangles. The three dimensional analog of this, cutting equal volume polyhedra into identical sets of tetrahedra is known as Hilbert's 3rd problem and is somewhat more involved. Around 1960, it was proven that any polygon can be triangulated using only acute triangles (all angles less than 90 degrees). Such triangulations are important in applications to numerical analysis and computer graphics, and practical applications lead to consideration of computational complexity (i.e., how fast can a "good" triangulation be computed?). For simple planar polygons, it was proven in the 1990's that the work needed is proportional to the number of sides of the polygon (linear complexity). However, for general polygon regions (allowing holes), a rigorous polynomial time bound was only proven about ten years ago, and the sharp polynomial power remains unknown. If time permits, I will discuss even more recent developments involving optimal angle bounds, 3-dimensional analogs, and connections to other parts of mathematics.