EQUILATERAL TRIANGULATIONS OF RIEMANN SURFACES

CHRISTOPHER BISHOP (STONY BROOK)

Abstract. After defining harmonic measure on a planar domain, I will discuss "true trees". These are trees drawn in the plane so that every edge has equal harmonic measure, and so that the measure restricted to each edge is the same from either side of the edge. Such trees correspond in a precise way to polynomials that have exactly two critical values and they form a special case of a dessin in Grothendieck's theory of *dessins d'enfant*, where a graph on a topological surface induces a conformal structure on that surface. I will explain the connection between dessins, equilateral triangulations of Riemann surfaces, and branched coverings of the 2-sphere (Belyi's theorem). Although it is easy to see that only countably many compact Riemann surfaces have any equilateral triangulation, Lasse Rempe and I recently proved that every non-compact surface has uncountably many. In particular, this implies that every Riemann surface is a holomorphic branched cover of the 2-sphere, branched over finitely many points. If time permits, I will briefly discuss "infinite true trees" that correspond to entire functions with exactly two critical values. A new, flexible construction of such functions, called "quasiconformal folding", has had numerous applications in complex analysis and holomorphic dynamics.